ANALYSIS OF THE EFFECTIVE HEAT CONDUCTION OF HONEYCOMB SANDWICH PANELS

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A method of analyzing the effective heat conduction of sandwich panels with different kinds of filler cells is elucidated.

Computational dependence to determine the effective heat conduction of a multilayer panel with honeycomb core of hexagonal cells are obtained in [1] on the basis of a theory of generalized conductivity. The computational model and the coupling equation to determine the heat conduction of the whole panel evidently change as the core cell shape and the quantity of layers vary. We present a general approach to finding to effective heat conductivity coefficients of a sandwich panel with different kinds of core cells.

Let us consider a sandwich panel (Fig. 1) containing two outer shells 3 and 4 and a honeycomb core whose cells are formed by corrugated tapes 1 fastened together and filled with a heat-insulating material 2. The cell shape is determined by the kind of corrugated tape whose profile can be a sinusoid (Fig. 2a), triangle (Fig. 2b), or trapezoid (Fig. 2c) formed by the arcs of semicircles (Fig. 2d). The corrugated tape is a periodic function $y = f(x)$, whose fundamental geometric parameters are the amplitude H, the spacing a, and the tape thickness δ_1.

The most complex problem is that of determining the heat conductivity of the honeycomb core in the direction of the principle axes of anisotropy x and y. We represent the honeycomb layer as a binary mixture of corrugated tape and cell core. We isolate a cell element of length $a/4$ for each kind of corrugated tape type (Fig. 2).

The heat conduction coefficient in the y direction on each section dx of the isolated cell is determined from the formula [2]

$$\bar{\lambda}_{12y} = \lambda_\tau \sin^2\beta + \lambda_n \cos^2\beta, \tag{1}$$

where λ_τ, λ_n are the principle heat-conduction coefficients in the tangential τ and normal n directions, determined from dependences for plates along and across the heat flux, respectively:

$$\lambda_\tau = \lambda_1 m_1 + \lambda_2 (1 - m_1); \tag{2}$$

$$\lambda_n = \frac{\lambda_2}{1 - m_1 (1 - \lambda_2/\lambda_1)}. \tag{3}$$

Let us convert (1) to a more convenient form for subsequent utilization by replacing tan β by the derivative dy/dx

$$\bar{\lambda}_{12y} = \frac{\lambda_\tau \operatorname{tg}^2\beta + \lambda_n}{1 + \operatorname{tg}^2\beta} = \frac{\lambda_n + \lambda_\tau (dy/dx)^2}{1 + (dy/dx)^2}.$$

The mean value of the heat conductivity in the segment $[x_1; x_2]$ is determined by integrating the last expression

$$\bar{\lambda}_{12y} = \frac{1}{x_2 - x_1}\int_{x_1}^{x_2} \lambda_y(x)\,dx = \frac{1}{x_2 - x_1}\int_{x_1}^{x_2} \frac{\lambda_n + \lambda_\tau (dy/dx)^2}{1 + (dy/dx)^2}\,dx = \lambda_\tau + \frac{\lambda_n - \lambda_\tau}{x_2 - x_1}\int_{x_1}^{x_2} \frac{dx}{1 + (dy/dx)^2}. \tag{4}$$

From analogous reasoning we find the mean value of the heat conductivity in the x direction in the segment $[y_1; y_2]$:

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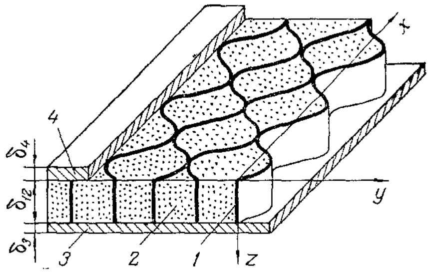

Fig. 1. Diagram of a honeycomb sandwich panel.

$$\bar{\lambda}_{12x} = \lambda_\tau + \frac{\lambda_n - \lambda_\tau}{y_2 - y_1} \int\limits_{y_1}^{y_2} \frac{dy}{1 + (dx/dy)^2}. \tag{5}$$

By knowing the specific form of the function describing the corrugated tape, we can determine the effective heat conductivity of the core layer along the x and y axes from (4) and (5).

We evidently obtain for a core from corrugated tape of triangular profile

$$\bar{\lambda}_{12y} = \lambda_\tau \sin^2\beta + \lambda_n \cos^2\beta; \tag{6}$$

$$\bar{\lambda}_{12x} = \lambda_\tau \cos^2\beta + \lambda_n \sin^2\beta, \tag{7}$$

where the volume concentration of tape in the cell is

$$m_1 = \frac{\delta_1 k}{H + \delta_1/\cos\beta} = \frac{1}{1 + H/\delta_1 k}; \quad k = 1/\cos\beta = \sqrt{1 + 4(H/a)^2}. \tag{8}$$

The heat conductivity of a cell with a corrugated tape of trapezoidal profile (Fig. 2c) is comprised of the heat conductivities of the two sections b and c with their volume content in the cell and their diagrams in parallel to the y direction and in series in the x direction taken into account

$$\bar{\lambda}_{12y} = \frac{\bar{\lambda}_y^b b}{b + c} + \frac{\bar{\lambda}_y^c c}{b + c}; \tag{9}$$

$$\bar{\lambda}_{12x} = \frac{b + c}{b/\bar{\lambda}_x^b + c/\bar{\lambda}_x^c}, \tag{10}$$

where

$$\bar{\lambda}_y^b = \frac{\lambda_2}{1 - m_1^b(1 - \lambda_2/\lambda_1)}; \quad \bar{\lambda}_y^c = \lambda_\tau^c \sin^2\beta + \lambda_n^c \cos^2\beta;$$

$$\bar{\lambda}_x^b = \lambda_1 m_1^b + \lambda_2(1 - m_1^b); \quad \bar{\lambda}_x^c = \lambda_\tau^c \cos^2\beta + \lambda_n^c \sin^2\beta;$$

$$m_1^b = \frac{b}{(1 + H/\delta_1)(b + c)}; \quad m_1^c = \frac{c}{(1 + H/\delta_1 k)(b + c)};$$

$$k = 1/\cos\beta = \sqrt{1 + H^2/4c^2}.$$

Values of λ_τ^c and λ_n^c in the section c are found from dependences (3) and (4) taking account of the concentration m_1^c.

We find the effective heat conductivity of a cell from a corrugated tape in the form of a semicircular arc (Fig. 2d) by substituting the values of the derivative dy/dx and dx/dy from the equation of a circle $x^2 + y^2 = R^2$ in (4) and (5):

$$\bar{\lambda}_{12y} = \lambda_\tau + \frac{\lambda_n - \lambda_\tau}{R} \int\limits_0^R \frac{dx}{1 + x^2/(R^2 - x^2)} = \frac{1}{3}\lambda_\tau + \frac{2}{3}\lambda_n; \tag{11}$$

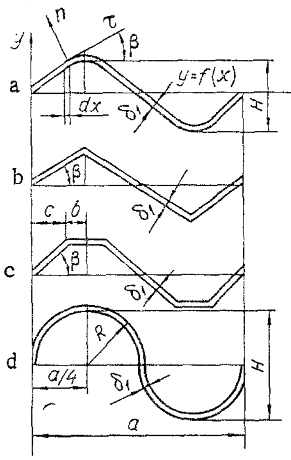

Fig. 2

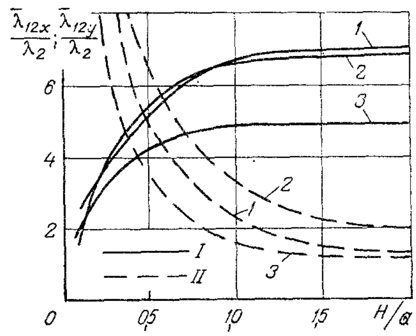

Fig. 3

Fig. 2. Profile of the corrugated tape in the core cell: a) sinusoid; b) triangle; c) trapezoid; and d) formed by arcs of semicircles.

Fig. 3. Dependence of the core layer heat conductivity on the amplitude of the corrugated tape ($\bar{\lambda}_{12y}/\lambda_2$ (I); $\bar{\lambda}_{12x}/\lambda_2$ (II); λ_1/λ_2 = 100; a = 60 mm; δ_1 = 2 mm): 1) triangular tape profile; 2) sinusoidal; 3) trapezoidal.

$$\bar{\lambda}_{12x} = \lambda_\tau + \frac{\lambda_n - \lambda_\tau}{R} \int\limits_0^R \frac{dy}{1 + y^2/(R^2 - y^2)} = \frac{1}{3}\lambda_\tau + \frac{2}{3}\lambda_n,$$

i.e., the heat conductivity of the cell is identical in the x and y directions, and the tape concentration in the cell is $m_1 = \pi\delta_1/2R$. Finally, we find the heat conductivity of a cell from corrugated tape described by a sinusoidal dependence:

$$y = 0.5H \sin \omega x. \tag{12}$$

The derivative of this function is

$$dy/dx = 0.5H\omega \cos \omega x = \bar{H} \cos \omega x.$$

Substituting this latter expression into (4), we obtain

$$\bar{\lambda}_{12y} = \lambda_\tau + \frac{\lambda_n - \lambda_\tau}{\omega a/4} \int\limits_0^{a/4} \frac{d\omega x}{1 + \bar{H}^2 \cos^2 \omega x} =$$

$$\lambda_\tau + \frac{\lambda_n - \lambda_\tau}{\pi/2} \frac{1}{\sqrt{1 + \bar{H}^2}} \operatorname{arctg} \frac{\operatorname{tg} \omega x}{\sqrt{1 + \bar{H}^2}} \Bigg|_0^{a/4} \to \tag{13}$$

$$\bar{\lambda}_{12y} = \lambda_\tau \left(1 - \frac{1}{\sqrt{1 + \bar{H}^2}}\right) + \frac{\lambda_n}{\sqrt{1 + \bar{H}^2}}.$$

We find the heat conductivity of the cell in the x direction analogously

$$\bar{\lambda}_{12x} = \frac{\lambda_\tau}{\sqrt{1 + \bar{H}^2}} + \lambda_n \left(1 - \frac{1}{\sqrt{1 + \bar{H}^2}}\right), \tag{14}$$

where λ_τ and λ_n are determined from (2) and (3) in which the tape concentration is

$$m_1 = \frac{k}{1 + H/\delta_1}. \tag{15}$$

As in the preceding expressions, the parameter k is the ratio between the arc length of the sinusoid in the cell element l_s and the cell length $a/4$. It should be noted that there is no exact analytical dependence to determine the arc length of the sinusoid, hence, it must be found by numerical integration of the expression:

$$l_s = \int_0^{a/4} \sqrt{1 + \bar{H}^2 \cos^2 \omega x}\, dx.$$

Computation of l_s on an electronic digital computer by the Simpson formula permitted obtaining the following approximate expression for the parameter k:

$$k = \sqrt{1 + 4.2\,(H/a)^2}\ .$$

The heat conductivity of the core layer in the z direction is defined for all kinds of corrugated tape as the heat conductivity of plates parallel to the heat flux:

$$\bar{\lambda}_{12z} = \lambda_1 m_1 + \lambda_2 (1 - m_1), \tag{16}$$

where the tape volume concentration m_1 is found from the dependence presented above for each type of corrugated tape.

The computational dependences of the relative effective heat conductivity of the core layer $\bar{\lambda}_{12y}/\lambda_2$ and $\bar{\lambda}_{12x}/\lambda_2$ on the relative amplitude of the corrugated tape H/a are represented in Fig. 3 for the three tape types considered for δ_1 = 2 mm; a = 60 mm; λ_1/λ_2 = 100. It is seen from the figure that the heat conductivity of the core layer changes sharply for $H/a < 1$, and the core heat conductivity varies slightly as the tape amplitude increases further. The heat conductivities $\bar{\lambda}_{12y}/\lambda_2$ of cores with triangular and sinusoidal profile tapes differ by not more than 10% for $H/a > 0.2$. In the domain $H/a = 0.5$ the heat conductivities of the core layer are similar in the x and y directions for each kind of cell, while they differ severalfold for values of $H/a > 1$. Therefore, by varying the structural geometric tape parameters, the relationship between the heat conductivities of the honeycomb core in the orthogonal x and y directions can be changed substantially.

The effective heat conductivity of the whole panel in the three coordinates can be found by knowing the heat conductivity of the honeycomb core and the shell. The shell and honeycomb core layers are in parallel in the x and y directions. Therefore, the effective heat conduction of the panel is

$$\bar{\lambda}_y = \frac{1}{\delta}(\lambda_4\delta_4 + \bar{\lambda}_{12y}\delta_{12} + \lambda_3\delta_3); \tag{17}$$

$$\bar{\lambda}_x = \frac{1}{\delta}(\lambda_4\delta_4 + \bar{\lambda}_{12x}\delta_{12} + \lambda_3\delta_3). \tag{18}$$

The panel layers in the z direction are connected in series, and the thermal resistivities of the layers are added

$$\frac{\delta}{\bar{\lambda}_z} = \frac{\delta_4}{\lambda_4} + \frac{\delta_{12}}{\bar{\lambda}_{12z}} + \frac{\delta_3}{\lambda_3}, \tag{19}$$

from which the effective panel heat conductivity is

$$\bar{\lambda}_z = \frac{\delta}{\delta_4/\lambda_4 + \delta_{12}/\bar{\lambda}_{12z} + \delta_3/\lambda_3}. \tag{20}$$

Therefore, the dependences obtained permit determination of the effective heat conductivity of honeycomb sandwich panels with different kinds of corrugated core in three mutually perpendicular directions.

NOTATION

l_s, length of sinusoid arc; δ, thickness of sandwich panel elements; a, H, corrugated tape spacing and amplitude, respectively; b, c, tape section lengths for trapezoidal profile; R, radius; β, angle; m, volume concentration of components; k, ratio between tape length in a cell element and cell length; $\bar{H} = \pi H/a$; $\omega = 2\pi/a$; λ, heat-conduction coefficient of a component; $\bar{\lambda}$, effective heat conduction of a component; λ_τ, λ_n, principal heat-conduction coefficients. Subscripts: 1, tape; 2, cell core material; 3, 4, panel shells; 12, layer of honeycomb panel core; b, c, sections of trapezoidal tape profile; x, y, z in the x, y, z directions.

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